(b) State and prove Jensen's formula.

10. (a) State Hadamard's factorization theorem and use it to show that

$$\sin \pi z = \prod_{n=1}^{\infty}\left(1-\frac{z^2}{n^2}\right)$$

(b) State and prove the Great Picard theorem.

Roll No. .... Total Pages : 4

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DMDE/M–19

COMPLEX ANALYSIS

Paper–MM–404

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. All questions carry equal marks.

UNIT–I

1. (a) Define complex line integral. State Cauchy's integral theorem. Verify Cauchy's theorem for the function $f(z) = z^3 - iz^2 - 5z + 2i$ if path is circle given by $|z - 1| = 2$.

(b) Define path in a region, bounded variation and simply connected domain. Also define winding number of a closed curve with simple properties.

2. State and prove Cauchy's integral formula for higher derivatives. Also evaluate $\int_C \frac{\sin z dz}{\left(z-\frac{\pi}{4}\right)^3}$ where C is the circle $\left|z-\frac{\pi}{4}\right| = \frac{1}{2}$.

3. (a) State and prove Liouville's theorem.

(b) If the function f(z) is analytic and one valued in $|z - a| < R$, prove that for $0 < r < R$, $f'(a) = \frac{1}{\pi r}\int_0^{2\pi} P(\theta)e^{-i\theta}d\theta$ where $P(\theta)$ is the real part of $f(a + re^{i\theta})$.

UNIT–II

4. (a) State Laurent's theorem and prove the uniqueness of it.

(b) State and prove Rouche's theorem.

5. (a) Define residue at infinity and evaluate

$$\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}.$$

(b) Prove that at each point z of a domain where f(z) is analytic and $f'(z) \neq 0$, the mapping w = (z) is conformal.

UNIT–III

6. (a) State and prove Hurwitz's theorem.

(b) State and prove Riemann mapping theorem.

7. (a) Define Gamma function and prove that

$$\Gamma(z)\,\Gamma(1-z) = \frac{\pi}{\sin \pi z}.$$

(b) State and prove Mittag-Leffler's theorem.

8. (a) Define analytic continuation and explain power series method for the analytic continuation.

(b) State and prove Schwarz's reflection principle.

UNIT–IV

9. (a) Let G and R be regions such that there is one-one analytic function F of G onto R. Let $a \in G$ and $\alpha = f(a)$. If g_a and γ_α are the Green's functions for G and R with singularities a and α respectively then show that $g_a(z) = \gamma_\alpha(f(z))$.